

Bouncing cosmology without anisotropy

Kh. Saaidi¹, S. Ziaee²

*Department of Physics, Faculty of Science, University of Kurdistan,
Sanandaj, Iran*

Abstract

Using non-linear equation of state for pressure and density energy, we show that the universe is began with a smooth and isotropic bounce. We use a non-linear equation of state which is a binary mixture of perfect fluid and dark energy. We show that in order to preserve a smooth and isotropic bounce, the source for contraction before the bounce, must have an equation of state with $\omega > 1$ (Ekpyrotic matter) and a dark energy with positive pressure.

Keywords: Anisotropic universe; Bouncing Cosmology; Early Universe.

¹ksaaidi@uok.ac.ir

²S.ziaee@uok.ac.ir

1 Introductions

The standard cosmological model furnishes an accurate description of the evolution of the universe, since 14 billion years ago approximately [1]. Regardless of its success, the standard cosmological model suffers from a few problems such as the initial singularity, the cosmological horizon, the flatness problem, the baryon asymmetry, the homogeneity problem, the large scale structure problem, and the nature of dark matter and dark energy [2]. Although inflation partially answer some problem, it does not solve the essential problem of the initial singularity [3]. The existence of an initial singularity is perturbing. Singularity can be naturally considered as a source of lawlessness [4], because the spacetime explanation breaks down there, and physical laws presuppose spacetime.

In the begining of the 80's, it was clear that the standard cosmological model was in crisis. The existence of the inflationary theory gave an answer to some of these problems and opened the window for a description of the origin of the spectrum of primeval fluctuations. In fact, inflation predicts the appearance of quantum fluctuations in the initial vacuums state, arrives to primeval perturbations seeding the observed cosmic large-scale structures [5]. These primeval fluctuation are endowed with a nearly scale-invariant spectrum, in agreement with observation of the cosmic microwave background [6]. It is well know that inflation produce an explanation for the homogeneity, flatness and horizon problem of the standard hot Big Bang cosmology. However, in spite of its successes, the theory of inflation does not solve the problems of the initial singularity and can not embedding inflation within a quantum gravity.

Mainly inspired by the string motivated pre-Big Bang scenarios [7, 8], bouncing models [9, 10, 11, 12, 13], i.e. models in which the universe undergoes a phase of contraction followed by expansion, have been proposed as alternatives to the inflationary paradigm [14].

The difficulties of embedding inflation within a quantum gravity theory and the persistence of the initial singularity in the inflationary scenario have motivated several proposals of alternative cosmologies. There is a general consensus on the existence of a high energy cut-off at the order of the Planck scale, at which classical general relativity should be replaced by a quantum gravity theory. From this point of view, the Big Bang singularity just represents the outcome of the extrapolation of general relativity beyond its domain of applicability, whereas the quantum gravity theory should regularize this singularity, replacing it by a maximum in the curvature and energy density of the universe. The existence of a contraction phase be-

fore the Big Bang has been argued in several frameworks. Following this hypothesis, the universe should contract from initial conditions in a low energy regime, evolving into a phase of higher and higher curvature, until the high energy cut-off of the true quantum gravity theory comes into play. This reverses the contraction into a standard decelerated expansion, thus avoiding the general relativistic singularity and replacing it by a cosmic bounce [24].

Bouncing cosmology model can simply solve the problems of flatness and horizon from standard cosmology model, but anisotropy in contraction phase is troublesome when the contribution phase is immaterial, anisotropy become quickly dominant and leads to a *velocity dominance singularity* [15, 16]. This typical result of general relativity can only be avoided if energy density of matter source growth more quickly than anisotropy. On the other hand, all sources with ω smaller than one growth very slowly and at the end they become anisotrop. When this occur, a mixmaster theory takes place with the development of chaotic *Belinskii, Khalatnikov and Lifshitz (BKL)* oscillations in the scale factors [15, 16, 17, 18]. Therefore, there is a probability that cross universe after bounce is unreliable anisotrop. According to this point that when strong curvature effect is dominant, mixmaster treatment takes place in high energy phase, so they rise this probability to solve anisotropy in contraction phase problem by adding a non-linear term to equation of state (EoS) [24].

In this paper we investigated the effects of a general non-linear term of EoS in the bouncing cosmological model. In order to understand this context the non-linear EoS can isotopize the universe at early times and at high energy regime, when the bounce is approached. We wanted to study on the possible use of a non-linear EoS as an effective way of representing a dark energy, to solve the anisotropy problem in contraction phase. Also, here we obtain the density energy, ρ , and anisotropic factor, σ^2 , for case that the EoS is a binary mixture of perfect fluid and dark energy.

2 Non-linear EoS

We assume that gravity in the contraction regime is determinate by Einstein equation. And also we suppose that the contraction regime is dominated by a binary mixture of a perfect fluid and dark energy with energy density ρ and pressure P . By $P_m = \omega\rho$ and a dark energy component with non-linear

form of ρ . We consider general form of a non-linear EoS as follows

$$P = P_m + P_d = \omega\rho + \epsilon \frac{\rho^\alpha}{\rho_c^{(\alpha-1)}}, \quad (1)$$

where ω is a constant value (is a pure number), that indicates the low energy EoS of the fluid, $\rho_c > 0$ is the transition scale and ϵ is the sign of the non-linear term. Here we will alone focus on the case $\epsilon > 0$, where it means that the pressure of dark energy is positive.

The easiest way to investigate the behavior of anisotropy in the contraction phase, is using from Bianchi type I models. The Bianchi type I models are a subclass of the Bianchi class A models. These models are homogeneous and anisotropic cosmological models including the flat Fridmann model [19]. The Bianchi type I cosmology can be defined by the Hubble expansion scaler and the tracefree shear tensor $\sigma_{\mu\nu}$, in which $\mu, \nu = 1, \dots, 3$ and $\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu}$. The energy-momentum tensor is give by

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu - P g_{\mu\nu}, \quad (2)$$

where ρ is the energy density, P the pressure and u is the 4-vector velocity. The energy conservation equation for a cosmological model including a perfect fluid is

$$\dot{\rho} = -3H(\rho + P), \quad (3)$$

where H is the Hubble parameter, i.e. $H = \frac{\dot{a}}{a}$. Using the Einstein equations and the Bianchi type I models with assume $\frac{8\pi G}{c^4} = 1$, we can obtain

$$H^2 = \frac{1}{3}(\rho + \sigma^2), \quad (4)$$

$$\dot{H} = -H^2 - \frac{1}{6}((\rho + 3P) + 4\sigma^2), \quad (5)$$

$$\dot{\sigma} = -3H\sigma. \quad (6)$$

The energy conservation (3) and non-linear EoS (1) can determine the energy density as a function of scale factor.

$$\rho = \rho_c \left[\frac{(1 + \omega)A^{(\alpha-1)}}{a^{3(1+\omega)(\alpha-1)} - \epsilon A^{(\alpha-1)}} \right]^{\frac{1}{\alpha-1}}, \quad (7)$$

$$A = \frac{\rho_0 a_0^{3(1+\omega)}}{[(1 + \omega)\rho_c^{(\alpha-1)} + \epsilon \rho_0^{(\alpha-1)}]^{\frac{1}{\alpha-1}}}, \quad (8)$$

where ρ_0 and a_0 express the energy density and scale factor at an arbitrary time t_0 . This is acceptable for all values of ϵ , ρ_0 and ω except for $\omega = -1$ [19]. Defining $a_* = |A|^{\frac{1}{3(1+\omega)}}$ and supposing $\rho > 0$, we arrive at

$$\rho = \rho_c \left[\frac{(1+\omega)}{\left(\frac{a}{a_*}\right)^{3(1+\omega)(\alpha-1)} - \epsilon} \right]^{\frac{1}{\alpha-1}}. \quad (9)$$

Using Eq.(6), we can obtain shear scales as a function of the scale factor as

$$\sigma^2 = \sigma_i^2 \left(\frac{a}{a_i} \right)^{-6}. \quad (10)$$

According to above equation, the shear growing larger for smaller a in the past and quickly decays at late times.

Note that in a contraction phase driven by a source fulfilling the energy conditions, hence we limited our work to the state $(1+\omega) > 0$.

The quantity of a_* can be obtained from the initial conditions of the universe, so if the universe begins with a scale factor a_i we arrive at

$$a_* = a_i \left[(1+\omega) \left(\frac{\rho_c}{\rho_i} \right)^{(\alpha-1)} + \epsilon \right]^{-\frac{1}{3(1+\omega)(\alpha-1)}}. \quad (11)$$

As a result of the existence of a contraction phase before the Big Bang that has been discussed in several articles [20, 21, 22, 23]. Our universe contract from initial conditions in a low energy phase, then larger and larger energy scales until it get to planck scale. In this case, the existence of a high energy cut-off at the order of the Planck scale and classical general relativity replaced by a quantum gravity theory. Therefore, when ρ reaches ρ_M quantum gravity theory comes into play and in this stage the contraction is stopped and the universe goes towards a standard decelerated expansion phase from a bounce. So that we defined the scale factor of the universe in the beginning of the bounce as

$$a_M = a_* \left[(1+\omega) \left(\frac{\rho_c}{\rho_M} \right)^{(\alpha-1)} + \epsilon \right]^{\frac{1}{3(1+\omega)(\alpha-1)}}. \quad (12)$$

One can estimate the anisotropic behavior of the universe from comparing the contribution of shear term with respect to matter term in Eq.(4), where σ term caused by space anisotropy. Then to avoiding an anisotropy

approach to the bounce this term should be very smaller than the matter term at the onset the bounce. Therefore, using Eqs.(9-12), we can arrive at

$$\frac{\sigma_M^2}{\rho_M} = \frac{\sigma_i^2}{\rho_M} \left[\frac{\left(\frac{\rho_c}{\rho_i}\right)^{(\alpha-1)} + \frac{\epsilon}{1+\omega}}{\left(\frac{\rho_c}{\rho_M}\right)^{(\alpha-1)} + \frac{\epsilon}{1+\omega}} \right]^{\frac{2}{(1+\omega)(\alpha-1)}}. \quad (13)$$

3 Typical example

In this section we want study some example which EoS of them are non-linear for considering the anisotropic behavior of the universe.

3.1 The EoS with a quadratic term

For $\alpha = 2$, Eq.(1) shows the EoS with a quadratic term as

$$P = \omega\rho + \epsilon \frac{\rho^2}{\rho_c}. \quad (14)$$

this form of non-linear EoS is studied in [24]. By the way, from Eqs.(9,11,12), we have

$$\rho = \frac{(1+\omega)\rho_c}{\left(\frac{a}{a_*}\right)^{3(1+\omega)} - \epsilon}, \quad (15)$$

$$a_* = a_i \left[(1+\omega) \left(\frac{\rho_c}{\rho_i} \right) + \epsilon \right]^{-\frac{1}{3(1+\omega)}}, \quad (16)$$

$$a_M = a_* \left[(1+\omega) \left(\frac{\rho_c}{\rho_M} \right) + \epsilon \right]^{\frac{1}{3(1+\omega)}}. \quad (17)$$

In this case, according to Eq.(15) to satisfy the assumption $\rho > 0$, a should satisfy $a_* < a < \infty$. Using Eqs.(15-17) and (13) with imposing the hierarchy $\rho_M \gg \rho_c \gg \rho_i$ for anisotropy fraction, we can obtain

$$\frac{\sigma_M^2}{\rho_M} \simeq \frac{\sigma_i^2}{\rho_i} \left(\frac{\rho_c}{\rho_i} \right)^{\frac{1-\omega}{1+\omega}} \left(\frac{\rho_c}{\rho_M} \right). \quad (18)$$

Eqs.(15-18) show that these result is in agreement with the obtained results in [24].

3.2 Modified Polytropic Like Gas

For $\alpha = 1 + \frac{1}{n}$, Eq.(1) shows the EoS of modified polytropic like gas, where n index is a positive ($n > 0$). So, from Eq.(9) we have

$$\rho = \rho_c \left[\frac{(1 + \omega)}{\left(\frac{a}{a_*}\right)^{\frac{3}{n}(1+\omega)} - \epsilon} \right]^n, \quad (19)$$

also, from Eqs.(11,12) we have

$$a_* = a_i \left[(1 + \omega) \left(\frac{\rho_c}{\rho_i} \right)^{\left(\frac{1}{n}\right)} + \epsilon \right]^{-\frac{n}{3(1+\omega)}}, \quad (20)$$

$$a_M = a_* \left[(1 + \omega) \left(\frac{\rho_c}{\rho_M} \right)^{\left(\frac{1}{n}\right)} + \epsilon \right]^{\frac{n}{3(1+\omega)}}. \quad (21)$$

In this case, according to Eq.(19) and to satisfy the assumption $\rho > 0$, a should satisfy $a_* < a < \infty$.

Now we like consider anisotropy behavior for the EoS of modified polytropic like gas. Using Eqs.(19-21) and (13) with imposing the hierarchy $\rho_M \gg \rho_c \gg \rho_i$, we can obtain

$$\frac{\sigma_M^2}{\rho_M} \simeq \frac{\sigma_i^2}{\rho_i} \left(\frac{\rho_c}{\rho_i} \right)^{\frac{1-\omega}{1+\omega}} \left(\frac{\rho_c}{\rho_M} \right), \quad (22)$$

where $\frac{\sigma_i^2}{\rho_i}$ is initial anisotropy, $\left(\frac{\rho_c}{\rho_i} \right)^{\frac{1-\omega}{1+\omega}}$ is growth factor that is arising from low energy phase, and $\frac{\rho_c}{\rho_M}$ is a reducing factor of anisotropy that is arising from high energy phase. According to Eq.(22) growth factor of anisotropy depends on the ω index that is the linear term coefficient in the EoS. When ω is more than one ($\omega > 1$), the growth factor is transformed to an additional reducing factor. As for Ekpyrotic matter (super-stiff matter), is $\omega > 1$.

By transition scale ρ_c is determined the efficiency of growth factor in the linear term and reducing factor in the non-linear term of EoS. If ρ value is very close to ρ_c , then just the growth factor caused by the linear term remains. While if ρ_c is very close to ρ_i , then only the reducing factor caused by the non-linear term remains.

3.3 Modified Chaplygin Like Gas

For $-1 < \alpha < 0$, Eq.(1) shows the EoS of modified chaplygin like gas. So, from Eq.(9) we have

$$\rho = \rho_c \left[\frac{\left(\frac{a_*}{a}\right)^{3(1+\omega)(1-\alpha)} - \epsilon}{(1+\omega)} \right]^{\frac{1}{1-\alpha}}, \quad (23)$$

Also from Eq.(11,12) we have

$$a_* = a_i \left[(1+\omega) \left(\frac{\rho_i}{\rho_c} \right)^{(1-\alpha)} + \epsilon \right]^{\frac{1}{3(1+\omega)(1-\alpha)}}, \quad (24)$$

$$a_M = a_* \left[(1+\omega) \left(\frac{\rho_M}{\rho_c} \right)^{(1-\alpha)} + \epsilon \right]^{-\frac{1}{3(1+\omega)(1-\alpha)}}. \quad (25)$$

In this case, according to Eq.(23) to satisfy the assumption $\rho > 0$, a should satisfy $0 < a < a_*$. Using Eqs.(23-25) and (13) with imposing the hierarchy $\rho_M \gg \rho_c \gg \rho_i$, we can obtain

$$\frac{\sigma_M^2}{\rho_M} \simeq \frac{\sigma_i^2}{\rho_i} \left(\frac{\rho_M}{\rho_c} \right)^{\frac{1-\omega}{1+\omega}} \left(\frac{\rho_i}{\rho_c} \right), \quad (26)$$

where $\frac{\sigma_i^2}{\rho_i}$ is initial anisotropy, $\left(\frac{\rho_M}{\rho_c} \right)^{\frac{1-\omega}{1+\omega}}$ is growth factor that is arising from high energy phase, and $\frac{\rho_i}{\rho_c}$ is a reducing factor of anisotropy that is arising from low energy phase. Similarly with the case before, the growth factor of anisotropy depends on the ω . Therefore, for $\omega > 1$ the growth factor is transformed to an additional reducing factor (in addition to $\frac{\rho_i}{\rho_c}$ term).

Note that in this case, if ρ_c is very close to the bounce scale ρ_M , then the growth factor shrinks to one and alone reducing factor caused by the linear term remains. While if transition scale ρ_c is very close to ρ_i , then the reducing factor disappears and alone the growth factor caused by the non-linear term of EoS remains.

Considering the Eqs.(18,22,26) the behavior of anisotropy surely depends on the initial amount of anisotropy $\frac{\sigma_i^2}{\rho_i}$ in the initial conditions. According to Eq.(18,22) if the amount of $\frac{\sigma_i^2}{\rho_i}$ is sufficiently low, we can have ρ_c relatively close to ρ_M then the effect of non-linear term is reduced. On the other hand, if the value of initial anisotropy is too high, then ρ_c should be very close to ρ_i . Therefore, it has an important role in reducing the anisotropy in this

case. According to Eq.(26), if the universe is previously fairly isotropic, we can have amount of ρ_c relatively close to ρ_i . Instead, if the universe begins in a very anisotropic case, therefore, for reducing the anisotropy, we should be take ρ_c very close to ρ_M , i.e. the effect of non-linear term in the EoS decreases.

In general, according to resulted equations, to preserve a smooth and isotropic bounce, the source of contraction should be a EoS with $\omega > 1$. Considering that linear EoS can not lonely solve the problem of anisotropic. Thus by addition general non-linear term to EoS and resulting a similar equations to (22) and (26) for $\omega > 1$ we can solve anisotropy problem in contraction phase. In fact, in the case of *ekpyrotic/cyclic* and *pre – Big Bang* models the initial expansion is only isotropic if $\omega > 1$ as in the case of general relativity [25].

4 conclusion

In this work, we have studied the early time behavior of anisotropy in contraction phase close to the bounce. Here we introduce a general non-linear EoS and investigated the behavior anisotropy of universe at early times and at high energy regime. Specially we solved some typical example and we have shown that to a smooth and isotropic bounce we must have a Ekpyrotic ($\omega > 1$) matter with a dark energy component with positive pressure at the onset of the bounce, which we called them generalized chaplygin like gas and polytropic like gas.

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